#### Section 8.2 Viscosity and Poiseuille's Law



# Physical basis of Hemodynamics

Velocity of the fluid

**FIGURE 8.2**  $\blacktriangleright$  Laminar flow. The length of the arrows indicates the magnitude of the velocity of the fluid.

#### 8.2 Viscosity and Poiseuille's Law

Frictionless flow is an idealization. In a real fluid, the molecules attract each other; consequently, relative motion between the fluid molecules is opposed by a frictional force, which is called *viscous friction*. Viscous friction is proportional to the velocity of flow and to the coefficient of viscosity for the given fluid. As a result of viscous friction, the velocity of a fluid flowing through a pipe varies across the pipe. The velocity is highest at the center and decreases toward the walls; at the walls of the pipe, the fluid is stationary. Such fluid flow is called *laminar*. Figure 8.2 shows the velocity profile for laminar flow in a pipe. The lengths of the arrows are proportional to the velocity across the pipe diameter.

If viscosity is taken into account, it can be shown (see reference [8-5]) that the rate of laminar flow Q through a cylindrical tube of radius R and length L is given by *Poiseuille's law*, which is

$$Q = \frac{\pi R^4 \left( P_1 - P_2 \right)}{8\eta L} \text{ cm}^3/\text{sec}$$
(8.6)

where  $P_1 - P_2$  is the difference between the fluid pressures at the two ends of the cylinder and  $\eta$  is the coefficient of viscosity measured in units of dyn (sec/cm<sup>2</sup>), which is called a *poise*. The viscosities of some fluids are listed in Table 8.1. In general, viscosity is a function of temperature and increases as the fluid becomes colder.

There is a basic difference between frictionless and viscous fluid flow. A frictionless fluid will flow steadily without an external force applied to it. This fact is evident from Bernoulli's equation, which shows that if the height and velocity of the fluid remain constant, there is no pressure drop along the flow path. But Poiseuille's equation for viscous flow states that a pressure

Fluid	Temperature (°C)	Viscosity (poise)
Water	20	0.01
Glycerin	20	8.3
Mercury	20	0.0155
Air	20	0.00018
Blood	37	0.04

# TABLE 8.1 ► Viscosities of Selected Fluids

drop always accompanies viscous fluid flow. By rearranging Eq. 8.6, we can express the pressure drop as

$$P_1 - P_2 = \frac{Q8\eta L}{\pi R^4}$$
(8.7)

The expression  $P_1 - P_2$  is the pressure drop that accompanies the flow rate Q along a length L of the pipe. The product of the pressure drop and the area of the pipe is the force required to overcome the frictional forces that tend to retard the flow in the pipe segment. Note that for a given flow rate the pressure drop required to overcome frictional losses decreases as the fourth power of the pipe radius. Thus, even though all fluids are subject to friction, if the area of the flow is large, frictional losses and the accompanying pressure drop are small and can be neglected. In these cases, Bernoulli's equation may be used with little error.

#### 8.3 Turbulent Flow

If the velocity of a fluid is increased past a critical point, the smooth laminar flow shown in Fig. 8.2 is disrupted. The flow becomes turbulent with eddies and whirls disrupting the laminar flow (see Fig. 8.3). In a cylindrical pipe the critical flow velocity  $v_c$  above which the flow is turbulent, is given by

$$v_c = \frac{\Re \eta}{\rho D} \tag{8.8}$$

Here *D* is the diameter of the cylinder,  $\rho$  is the density of the fluid, and  $\eta$  is the viscosity. The symbol  $\Re$  is the *Reynold's number*, which for most fluids has a value between 2000 and 3000. The frictional forces in turbulent flow are greater than in laminar flow. Therefore, as the flow turns turbulent, it becomes more difficult to force a fluid through a pipe.



**FIGURE 8.3** ► Turbulent fluid flow.

#### 8.4 Circulation of the Blood

The circulation of blood through the body is often compared to a plumbing system with the heart as the pump and the veins, arteries, and capillaries as the pipes through which the blood flows. This analogy is not entirely correct. Blood is not a simple fluid; it contains cells that complicate the flow, especially when the passages become narrow. Furthermore, the veins and arteries are not rigid pipes but are elastic and alter their shape in response to the forces applied by the fluid. Still, it is possible to analyze the circulatory system with reasonable accuracy using the concepts developed for simple fluids flowing in rigid pipes.

Figure 8.4 is a drawing of the human circulatory system. The blood in the circulatory system brings oxygen, nutrients, and various other vital substances to the cells and removes the metabolic waste products from the cells. The blood is pumped through the circulatory system by the heart, and it leaves the heart through vessels called *arteries* and returns to it through *veins*.

The mammalian heart consists of two independent pumps, each made of two chambers called the *atrium* and the *ventricle*. The entrances to and exits from these chambers are controlled by valves that are arranged to maintain the flow of blood in the proper direction. Blood from all parts of the body except the lungs enters the right atrium, which contracts and forces the blood into the right ventricle. The ventricle then contracts and drives the blood through the pulmonary artery into the lungs. In its passage through the lungs, the blood releases carbon dioxide and absorbs oxygen. The blood then flows into the left atrium via the pulmonary vein. The contraction of the left atrium forces the blood into the left ventricle, which on contraction drives the oxygen-rich blood through the aorta into the arteries that lead to all parts of the body except the lungs. Thus, the right side of the heart pumps the blood through the lungs, and the left side pumps it through the rest of the body.



**FIGURE 8.4** Schematic diagram showing various routes of the circulation.

The large artery, called the *aorta*, which carries the oxygenated blood away from the left chamber of the heart, branches into smaller arteries, which lead to the various parts of the body. These in turn branch into still smaller arteries, the smallest of which are called *arterioles*. As we will explain later, the arterioles play an important role in regulating the blood flow to specific regions in

the body. The arterioles branch further into narrow capillaries that are often barely wide enough to allow the passage of single blood cells.

The capillaries are so profusely spread through the tissue that nearly all the cells in the body are close to a capillary. The exchange of gases, nutrients, and waste products between the blood and the surrounding tissue occurs by diffusion through the thin capillary walls (see Chapter 9). The capillaries join into tiny veins called *venules*, which in turn merge into larger and larger veins that lead the oxygen-depleted blood back to the right atrium of the heart.

## 8.5 Blood Pressure

The contraction of the heart chambers is triggered by electrical pulses that are applied simultaneously both to the left and to the right halves of the heart. First the atria contract, forcing the blood into the ventricles; then the ventricles contract, forcing the blood out of the heart. Because of the pumping action of the heart, blood enters the arteries in spurts or pulses. The maximum pressure driving the blood at the peak of the pulse is called the *systolic pressure*. The lowest blood pressure between the pulses is called the *diastolic pressure*. In a young healthy individual the systolic pressure is about 120 torr (mm Hg) and the diastolic pressure is about 80 torr. Therefore the average pressure of the pulsating blood at heart level is 100 torr.

As the blood flows through the circulatory system, its initial energy, provided by the pumping action of the heart, is dissipated by two loss mechanisms: losses associated with the *expansion and contraction* of the arterial walls and *viscous friction* associated with the blood flow. Due to these energy losses, the initial pressure fluctuations are smoothed out as the blood flows away from the heart, and the average pressure drops. By the time the blood reaches the capillaries, the flow is smooth and the blood pressure is only about 30 torr. The pressure drops still lower in the veins and is close to zero just before returning to the heart. In this final stage of the flow, the movement of blood through the veins is aided by the contraction of muscles that squeeze the blood toward the heart. One-way flow is assured by unidirectional valves in the veins.

The main arteries in the body have a relatively large radius. The radius of the aorta, for example, is about 1 cm; therefore, the pressure drop along the arteries is small. We can estimate this pressure drop using Poiseuille's law (Eq. 8.7). However, to solve the equation, we must know the rate of blood flow. The rate of blood flow Q through the body depends on the level of physical activity. At rest, the total flow rate is about 5 liter/min. During intense activity the flow rate may rise to about 25 liter/min. Exercise 8-1 shows that at peak flow the pressure drop per centimeter length of the aorta



**FIGURE 8.5 •** Blood pressure in a reclining and in an erect person.

is only 42.5 dyn/cm<sup>2</sup> ( $3.19 \times 10^{-2}$  torr), which is negligible compared to the total blood pressure.

Of course, as the aorta branches, the size of the arteries decreases, resulting in an increased resistance to flow. Although the blood flow in the narrower arteries is also reduced, the pressure drop is no longer negligible (see Exercise 8-2). The average pressure at the entrance to the arterioles is about 90 torr. Still, this is only a 10% drop from the average pressure at the heart. The flow through the arterioles is accompanied by a much larger pressure drop, about 60 torr. As a result, the pressure at the capillaries is only about 30 torr.

Since the pressure drop in the main arteries is small, when the body is horizontal, the average arterial pressure is approximately constant throughout the body. The arterial blood pressure, which is on the average 100 torr, can support a column of blood 129 cm high (see Eq. 7.1 and Exercise 8-3). This means that if a small tube were introduced into the artery, the blood in it would rise to a height of 129 cm (see Fig. 8.5).

If a person is standing erect, the blood pressure in the arteries is not uniform in the various parts of the body. The weight of the blood must be taken into account in calculating the pressure at various locations. For example, the average pressure in the artery located in the head, 50 cm above the heart (see Exercise 8-4a) is  $P_{\text{head}} = P_{\text{heart}} - \rho gh = 61$  torr. In the feet, 130 cm below the heart, the arterial pressure is 200 torr (see Exercise 8-4b).

The cardiovascular system has various flow-control mechanisms that can compensate for the large arterial pressure changes that accompany shifts in the position of the body. Still, it may take a few seconds for the system to compensate. Thus, a person may feel momentarily dizzy as he/she jumps up from a prone position. This is due to the sudden decrease in the blood pressure of the brain arteries, which results in a temporary decrease of blood flow to the brain.

The same hydrostatic factors operate also in the veins, and here their effect may be more severe than in the arteries. The blood pressure in the veins is lower than in the arteries. When a person stands motionless, the blood pressure is barely adequate to force the blood from the feet back to the heart. Thus when a person sits or stands without muscular movement, blood gathers in the veins of the legs. This increases the pressure in the capillaries and may cause temporary swelling of the legs.

#### 8.6 Control of Blood Flow

The pumping action of the heart (that is, blood pressure, flow volume and rate of heart beat) is regulated by a variety of hormones. Hormones are molecules, often proteins, that are produced by organs and tissues in different parts of the body. They are secreted into the blood stream and carry messages from one part of the body to another. Hormones affecting the heart are produced in response to stimuli such as need for more oxygen, changes in body temperature, and various types of emotional stress.

The flow of blood to specific parts of the body is controlled by the arterioles. These small vessels that receive blood from the arteries have an average diameter of about 0.1 mm. The walls of the arterioles contain smooth muscle fibers that contract when stimulated by nerve impulses and hormones. The contraction of the arterioles in one part of the body reduces the blood flow to that region and diverts it to another. Since the radius of the arterioles is small, constriction is an effective method for controlling blood flow. Poiseuille's equation shows that if the pressure drop remains constant, a 20% decrease in the radius reduces the blood flow by more than a factor of 2 (see Exercise 8-5).

A stress-induced heart condition called stress cardiomyopathy (broken heart syndrome) has only recently been clearly identified by Western medicine. The syndrome occurs most frequently after a sudden intense emotional trauma such as death in the family, an experience of violence, or extreme anger. The symptoms are similar to an acute heart attack, but the coronary arteries are found to be normal and the heart tissue is not damaged. It has suggested that the condition is triggered by an excessive release of stress-related hormones called chatecholamines.

### 8.7 Energetics of Blood Flow

For an individual at rest, the rate of blood flow is about 5 liter/min. This implies that the average velocity of the blood through the aorta is 26.5 cm/sec (see Exercise 8-6). However, the blood in the aorta does not flow continuously. It moves in spurts. During the period of flow, the velocity of the blood is about three times as high as the overall average value calculated in Exercise 8-6. Therefore, the kinetic energy per cubic centimeter of flowing blood is

$$KE = \frac{1}{2}\rho v^2 = \frac{1}{2}(1.05) \times (79.5)^2 = 3330 \,\mathrm{erg/cm^3}$$

We mentioned earlier that energy density (energy per unit volume) and pressure are measured by the same unit (i.e.,  $1 \text{ erg/cm}^3 = 1 \text{ dyn/cm}^2$ ); therefore, they can be compared to each other. The kinetic energy of 3330 erg/cm<sup>3</sup> is equivalent to 2.50 torr pressure; this is small compared to the blood pressure in the aorta (which is on the average 100 torr). The kinetic energy in the smaller arteries is even less because, as the arteries branch, the overall area increases and, therefore, the flow velocity decreases. For example, when the total flow rate is 5 liter/min, the blood velocity in the capillaries is only about 0.33 mm/sec.

The kinetic energy of the blood becomes more significant as the rate of blood flow increases. For example, if during physical activity the flow rate increases to 25 liter/min, the kinetic energy of the blood is 83,300 erg/cm<sup>3</sup>, which is equivalent to a pressure of 62.5 torr. This energy is no longer negligible compared to the blood pressure measured at rest. In healthy arteries, the increased velocity of blood flow during physical activity does not present a problem. During intense activity, the blood pressure rises to compensate for the pressure drop.

#### 8.8 Turbulence in the Blood

Equation 8.8 shows that if the velocity of a fluid exceeds a specific critical value, the flow becomes turbulent. Through most of the circulatory system the blood flow is laminar. Only in the aorta does the flow occasionally become turbulent. Assuming a Reynold's number of 2000, the critical velocity for the onset of turbulence in the 2-cm-diameter aorta is, from Eq. 8.8,

$$V_c = \frac{\Re \eta}{\rho D} = \frac{2000 \times 0.04}{1.05 \times 2} = 38 \text{ cm/sec}$$

For the body at rest, the flow velocity in the aorta is below this value. But as the level of physical activity increases, the flow in the aorta may exceed the critical rate and become turbulent. In the other parts of the body, however, the flow remains laminar unless the passages are abnormally constricted.

Laminar flow is quiet, but turbulent flow produces noises due to vibrations of the various surrounding tissues, which indicate abnormalities in the circulatory system. These noises, called *bruit*, can be detected by a stethoscope and can help in the diagnosis of circulatory disorders.

#### 8.9 Arteriosclerosis and Blood Flow

Arteriosclerosis is the most common of cardiovascular diseases. In the United States, an estimated 200,000 people die annually as a consequence of this disease. In arteriosclerosis, the arterial wall becomes thickened, and the artery is narrowed by deposits called *plaque*. This condition may seriously impair the functioning of the circulatory system. A 50% narrowing (stenosis) of the arterial area is considered moderate. Sixty to seventy percent is considered severe, and a narrowing above 80% is deemed critical. One problem caused by stenosis is made clear by Bernoulli's equation. The blood flow through the region of constriction is speeded up. If, for example, the radius of the artery is narrowed by a factor of 3, the cross-sectional area decreases by a factor of 9, which results in a nine-fold increase in velocity. In the constriction, the kinetic energy increases by  $9^2$ , or 81. The increased kinetic energy is at the expense of the blood pressure; that is, in order to maintain the flow rate at the higher velocity, the potential energy due to pressure is converted to kinetic energy. As a result, the blood pressure in the constricted region drops. For example, if in the unobstructed artery the flow velocity is 50 cm/sec, then in the constricted region, where the area is reduced by a factor of 9, the velocity is 450 cm/sec. Correspondingly, the pressure is decreased by about 80 torr (see Exercise 8-8). Because of the low pressure inside the artery, the external pressure may actually close off the artery and block the flow of blood. When such a blockage occurs in the coronary artery, which supplies blood to the heart muscle, the heart stops functioning.

Stenosis above 80% is considered critical because at this point the blood flow usually becomes turbulent with inherently larger energy dissipation than is associated with laminar flow. As a result, the pressure drop in the situation presented earlier is even larger than calculated using Bernoulli's equation. Further, turbulent flow can damage the circulatory system because parts of the flow are directed toward the artery wall rather than parallel to it, as in laminar flow. The blood impinging on the arterial wall may dislodge some of the plaque deposit which downstream may clog a narrower part of the artery. If such clogging occurs in a cervical artery, blood flow to some part of the brain is interrupted causing an *ischemic stroke*.

There is another problem associated with arterial plaque deposit. The artery has a specific elasticity; therefore, it exhibits certain springlike properties. Specifically, in analogy with a spring, the artery has a natural frequency at which it can be readily set into vibrational motion. (See Chapter 5, Eq. 5.6.) The natural frequency of a healthy artery is in the range 1 to 2 kilohertz. Deposits of plaque cause an increase in the mass of the arterial wall and a decrease in its elasticity. As a result, the natural frequency of the artery is significantly decreased, often down to a few hundred hertz. Pulsating blood flow contains frequency components in the range of 450 hertz. The plaque-coated artery with its lowered natural frequency may now be set into resonant vibrational motion, which may dislodge plaque deposits or cause further damage to the arterial wall.

### 8.10 Power Produced by the Heart

The energy in the flowing blood is provided by the pumping action of the heart. We will now compute the power generated by the heart to keep the blood flowing in the circulatory system.

The power  $P_H$  produced by the heart is the product of the flow rate Q and the energy E per unit volume of the blood; that is,

$$P_H = Q\left(\frac{\mathrm{cm}^3}{\mathrm{sec}}\right) \times E\left(\frac{\mathrm{erg}}{\mathrm{cm}^2}\right) = Q \times E \mathrm{erg/sec}$$
 (8.9)

At rest, when the blood flow rate is 5 liter/min, or  $83.4 \text{ cm}^3/\text{sec}$ , the kinetic energy of the blood flowing through the aorta is  $3.33 \times 10^3 \text{ erg/cm}^3$ . (See previous section.) The energy corresponding to the systolic pressure of 120 torr is  $160 \times 10^3 \text{ erg/cm}^3$ . The total energy is  $1.63 \times 10^5 \text{ erg/cm}^3$ —the sum of the kinetic energy and the energy due to the fluid pressure. Therefore, the power *P* produced by the left ventricle of the heart is

$$P = 83.4 \times 1.63 \times 10^5 = 1.35 \times 10^7 \text{ erg/sec} = 1.35 \text{ W}$$

Exercise 8-9 shows that during intense physical activity when the flow rate increases to 25 liters/min, the peak power output of the left ventricle increases to 10.1 W.

#### Section 8.11 Measurement of Blood Pressure

The flow rate through the right ventricle, which pumps the blood through the lungs, is the same as the flow through the left ventricle. Here, however, the blood pressure is only one sixth the pressure in the aorta. Therefore, as shown in Exercise 8-10, the power output of the right ventricle is 0.25 W at rest and 4.5 W during intense physical activity. Thus, the total peak power output of the heart is between 1.9 and 14.6 W, depending on the intensity of the physical activity. While in fact the systolic blood pressure rises with increased blood flow, in these calculations we have assumed that it remains at 120 torr.

#### 8.11 Measurement of Blood Pressure

The arterial blood pressure is an important indicator of the health of an individual. Both abnormally high and abnormally low blood pressures indicate some disorders in the body that require medical attention. High blood pressure, which may be caused by constrictions in the circulatory system, certainly implies that the heart is working harder than usual and that it may be endangered by the excess load. Blood pressure can be measured most directly by inserting a vertical glass tube into an artery and observing the height to which the blood rises (see Fig. 8.5). This was, in fact, the way blood pressure was first measured in 1733 by Reverend Stephen Hales, who connected a long vertical glass tube to an artery of a horse. Although sophisticated modifications of this technique are still used in special cases, this method is obviously not satisfactory for routine clinical examinations. Routine measurements of blood pressure are now most commonly performed by the cut-off method. Although this method is not as accurate as direct measurements, it is simple and in most cases adequate. In this technique, a cuff containing an inflatable balloon is placed tightly around the upper arm. The balloon is inflated with a bulb, and the pressure in the balloon is monitored by a pressure gauge. The initial pressure in the balloon is greater than the systolic pressure, and the flow of blood through the artery is therefore cut off. The observer then allows the pressure in the balloon to fall slowly by releasing some of the air. As the pressure drops, she listens with a stethoscope placed over the artery downstream from the cuff. No sound is heard until the pressure in the balloon decreases to the systolic pressure. Just below this point the blood begins to flow through the artery; however, since the artery is still partially constricted, the flow is turbulent and is accompanied by a characteristic sound. The pressure recorded at the onset of sound is the systolic blood pressure. As the pressure in the balloon drops further, the artery expands to its normal size, the flow becomes laminar, and the noise disappears. The pressure at which the sound begins to fade is taken as the diastolic pressure.

In clinical measurements, the variation of the blood pressure along the body must be considered. The cut-off blood pressure measurement is taken with the cuff placed on the arm approximately at heart level.